6th Annual Lexington Math Tournament Guts Round - Part 1

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6th Annual Lexington Math Tournament Guts Round - Part 2

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6th Annual Lexington Math Tournament Guts Round - Part 3

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6th Annual Lexington Math Tournament Guts Round - Part 4

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6th Annual Lexington Math Tournament Guts Round - Part 5

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6th Annual Lexington Math Tournament Guts Round - Part 6

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6th Annual Lexington Math Tournament Guts Round - Part 7

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6th Annual Lexington Math Tournament Guts Round - Part 8

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6th Annual Lexington Math Tournament Guts Round - Part 9

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6th Annual Lexington Math Tournament Guts Round - Part 10

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6th Annual Lexington Math Tournament Guts Round - Part 11

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6th Annual Lexington Math Tournament Guts Round - Part 12

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6th Annual Lexington Math Tournament Guts Round - Part 1

1.[4] 1. Every angle of a regular polygon has degree measure 179.99 degrees. How many sides does it have?

2.[4] What is $\frac{1}{20} + \frac{1}{1} + \frac{1}{5}$?

3.[4] If the area bounded by the lines $y = 0$, $x = 0$, and $x = 3$ and the curve $y = f(x)$ is 10 units, what is the area bounded by $y = 0$, $x = 0$, $x = 6$, and $y = f(x/2)$?

6th Annual Lexington Math Tournament Guts Round - Part 2

4.[5] How many ways can 42 be expressed as the sum of 2 or more consecutive positive integers?

5.[5] How many integers less than or equal to 2015 can be expressed as the sum of 2 (not necessarily distinct) powers of two?

6.[5] p,q, and $q^2 - p^2$ are all prime. What is pq?

6th Annual Lexington Math Tournament Guts Round - Part 3

7.[6] Let $p(x) = x^2 + ax + a$ be a polynomial with integer roots, where a is an integer. What are all the possible values of a?

8.[6] In a given right triangle, the perimeter is 30 and the sum of the squares of the sides is 338. Find the lengths of the three sides.

9.[6] Each of the 6 main diagonals of a regular hexagon is drawn, resulting in 6 triangles. Each of those triangles is then split into 4 equilateral triangles by connecting the midpoints of the 3 sides. How many triangles are in the resulting figure?

6th Annual Lexington Math Tournament Guts Round - Part 4

10.[6] Let $f = 5x + 3y$, where x and y are positive real numbers such that xy is 100. Find the minimum possible value of f.

11.[6] An integer is called "Awesome" if its base 8 expression contains the digit string 17 at any point (i.e. if it ever has a 1 followed immediately by a 7). How many integers from 1 to 500 (base 10) inclusive are Awesome?

12.[6] A certain pool table is a rectangle measuring 15×24 feet, with 4 holes, one at each vertex. When playing pool, Joe decides that a ball has to hit at least 2 sides before getting into a hole or else the shot does not count. What is the minimum distance a ball can travel after being hit on this table if it was hit at a vertex (assume it only stops after going into a hole) such that the shot counts?

6th Annual Lexington Math Tournament Guts Round - Part 5

13.[7] Sally is at the special glasses shop, where there are many different optical lenses that distort what she sees and cause her to see things strangely. Whenever she looks at a shape through lens A, she sees a shape with 2 more sides than the original (so a square would look like a hexagon). When she looks through lens B, she sees the shape with 3 fewer sides (so a hexagon would look like a triangle). How many sides are in the shape that has 200 more diagonals when looked at from lense A than from lense B?

14.[7] How many ways can you choose 2 cells of a 5 by 5 grid such that they aren't in the same row or column?

15.[7] If $a + \frac{1}{b} = (2015)^{-1}$ and $b + \frac{1}{a} = (2016)^2$ then what are all the possible values of b?

6th Annual Lexington Math Tournament Guts Round - Part 6

16.[7] In Canadian football, linebackers must wear jersey numbers from $30 - 35$ while defensive linemen must wear numbers from $33 - 38$ (both intervals are inclusive). If a team has 5 linebackers and 4 defensive linemen, how many ways can it assign jersey numbers to the 9 players such that no two people have the same jersey number?

17.[7] What is the maximum possible area of a right triangle with hypotenuse 8?

18.[7] 9 people are to play touch football. One will be designated the quarterback, while the other eight will be divided into two (indistinct) teams of 4. How many ways are there for this to be done?

6th Annual Lexington Math Tournament Guts Round - Part 7

19.[8] Express the decimal 0.3 in base 7.

20.[8] 2015 people throw their hats in a pile. One at a time, they each take one hat out of the pile so that each has a random hat. What is the expected number of people who get their own hat?

21.[8] What is the area of the largest possible trapezoid that can be inscribed in a semicircle of radius 4?

6th Annual Lexington Math Tournament Guts Round - Part 8

22.[8] What is the base 7 expression of $1211_3 \cdot 1110_2 \cdot 292_{11} \cdot 20_3$?

23.[8] Let $f(x)$ equal the ratio of the surface area of a sphere of radius x to the volume of that same sphere. Let $g(x)$ be a quadratic polynomial in the form $x^2 + bx + c$ with $g(6) = 0$ and the minimum value of $g(x)$ equal to c. Express $g(x)$ as a function of $f(x)$ (e.g. in terms of $f(x)$).

24.[8] In the country of Tahksess, the income tax code is very complicated. Citizens are taxed 40% on their first $\$20,000$ and 45% on their next $\$40,000$ and 50% on their next $\$60,000$ and so on, with each 5% increase in tax rate affecting $\$20,000$ more than the previous tax rate. The maximum tax rate, however, is 90%. What is the overall tax rate (percentage of money owed) on 1 million dollars in income?

6th Annual Lexington Math Tournament Guts Round - Part 9

25.[9] For how many nonempty subsets of $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\}$ is the sum of the elements divisble by 32?

26.[9] America declared independence in 1776. Take the sum of the cubes of the digits of 1776 and let that equal S_1. Sum the cubes of the digits of S_1 to get S_2. Repeat this process 1776 times. What is S_{1776}?

27.[9] Every Golden Grahams box contains a randomly colored toy car, which is one of four colors.What is the expected number of boxes you have to buy in order to obtain one car of each color?

6th Annual Lexington Math Tournament Guts Round - Part 10

28. [10] Let B be the answer to Question 29 and C be the answer to Question 30. What is the sum of the square roots of B and C?

29. [10] Let A be the answer to Question 28 and C be the answer to Question 30. What is the sum of the sums of the digits of A and C?

30. [10] Let A be the answer to Question 28 and B be the answer to Question 29. What is $A + B$?

6th Annual Lexington Math Tournament Guts Round - Part 11

31. [11] If $x + \frac{1}{x} = 4$, find $x^6 + \frac{1}{x^6}$.

32. [11] Given a positive integer n and a prime p, there is are unique nonnegative integers a and b such that $n = p^b \cdot a$ and $\gcd(a, p) = 1$. Let $v_p(n)$ denote this uniquely determined a. Let S denote the set of the first 20 primes. Find $\sum_{p \in S} v_p(1 + \sum_{i=0}^{100} p^i)$.

33. [11] Find the maximum value of n such that $n + \sqrt{(n-1) + \sqrt{(n-2) + \cdots + \sqrt{1}}} < 49$ (Note: there would be $n - 1$ square roots and n total terms).

6th Annual Lexington Math Tournament Guts Round - Part 12

34.[≤ 20] Give two numbers a and b such that $2015^a < 2015! < 2015^b$. **If you are incorrect you get -5 points**; if you do not answer you get 0 points; otherwise you get $\max\{20 - 0.02(|b - a| - 1), 0\}$ points, rounded down to the nearest integer.

35.[≤ 20] Twin primes are prime numbers whose difference is 2. Let (a, b) be the 91717^{th} pair of twin primes, with $a < b$. Let $k = a^b$, and suppose that j is the number of digits in the base 10 representation of k. What is j^5? If the correct answer is n and you say m, you will receive $\max(20 - |\log(|\frac{m}{n}|)|, 0)$ points, rounded down to the nearest integer.

36.[$\leq K$] Write down any positive integer. Let the sum of the valid submissions (i.e. positive integer submissions) for all teams be S. One team will be chosen randomly, according to the following distribution: if your team's submission is n, you will be chosen with probability $\frac{n}{S}$. The amount of points that the chosen team will win is the greatest integer not exceeding $\min\{K, \frac{10000}{S}\}$. K is a predetermined secret value.